AB Calculus Quiz #14 v.U
Integration: U-Substitution
Dr. Wisniewski Spring 2020

Name Solution Bai'babu'   Period I

Instructions: Solve the problems below. Please show your work for partial credit and box or circle your answers. A calculator is NOT permitted on this portion of the quiz.

1. (2 Pts Each) Evaluate each of the following indefinite integrals.

a. $\int (t^2+5)^2 2t\,dt$ Let $u = t^2+5$, $du = 2t\,dt$

$\int u^2 du$

$= \frac{1}{3}u^3 + C$

$= \boxed{\frac{1}{3}(t^2+5)^3 + C}$

b. $\int x^3\sqrt{2x^4-1}\,dx$ let $u = 2x^4-1$, $du = 8x^3\,dx$

$\frac{1}{8}\int 8x^3\sqrt{2x^4-1}\,dx$

$= \frac{1}{8}\int u^{1/2}du = \frac{1}{8}\cdot\frac{2}{3}u^{3/2}+C = \frac{1}{12}u^{3/2}+C$

$= \boxed{\frac{1}{12}(2x^4-1)^{3/2}+C}$

c. $\int \frac{u}{\sqrt{1-u^2}}\,du$ let $w = 1-u^2$, $dw = -2u\,du$

$-\frac{1}{2}\int \frac{-2u\,du}{\sqrt{1-u^2}} = -\frac{1}{2}\int\frac{dw}{w^{1/2}} = -\frac{1}{2}\int w^{-1/2}dw = -\frac{1}{\cancel{2}}\cancel{2}w^{1/2}+C$

$= -w^{1/2}+C = \boxed{-\sqrt{1-u^2}+C}$ $-.5$ for missing neg sign.

d. $\int \cos^5 3x\,\sin 3x\,dx$ Let $u = \cos 3x$, $du = -3\sin 3x\,dx$

$-\frac{1}{3}\int \cos^5 3x\underbrace{(-3)\sin 3x\,dx}_{du}$

$= -\frac{1}{3}\int u^5 du = -\frac{1}{3}\cdot\frac{1}{6}u^6+C = -\frac{1}{18}u^6+C$

$= \boxed{-\frac{1}{18}\cos^6 3x + C}$

2. (2 Pts Each) Evaluate each of the flowing definite integrals.

a. $\int_{-\pi/4}^{\pi/4} \cos 2t\, dt$ $\overset{\text{even}}{=} 2\int_0^{\pi/4} \cos 2t\, dt$

let $u = 2t$
$du = 2dt$

$= \int_0^{\pi/2} \cos u\, du = \sin u \Big|_0^{\pi/2}$

when $t = \pi/4, u = \pi/2$
$t = 0, u = 0$

$= \sin \pi/2 - \sin 0$

1

b. $\int_0^1 xe^{-x^2} dx$

let $u = -x^2$
$du = -2x\,dx$

when $x = 0, u = 0$
$x = 1, u = -1$

$-\frac{1}{2}\int_0^{-1} -2xe^{-x^2} dx = -\frac{1}{2}\int_0^{-1} e^u du = \frac{1}{2}\int_{-1}^0 e^u du = \frac{1}{2} e^u \Big|_{-1}^0$

$= \frac{1}{2}\left[e^0 - e^{-1}\right] = \frac{1}{2}(1 - 1/e) = \boxed{\frac{1}{2}(1 - 1/e)}$

c. $\int_1^5 \frac{x}{\sqrt{2x+1}} dx$

let $u = 2x+1$
$du = 2dx$

when $x = 1, u = 3$
$x = 5, u = 11$

$u - 1 = 2x$
$x = \frac{1}{2}(u-1)$

$\frac{1}{2}\cdot\frac{1}{2}\int_3^{11} \frac{u-1}{u^{1/2}} 2\,dx = \frac{1}{4}\int_3^{11} \left(\frac{u-1}{u^{1/2}}\right) du = \frac{1}{4}\int_3^{11} (u^{1/2} - u^{-1/2}) du$

$= \frac{1}{4}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_3^{11} = \frac{2}{4}\left[\frac{1}{3}u^{3/2} - u^{1/2}\right]_3^{11}$ 0

$= \frac{1}{2}\left[\frac{1}{3}11^{3/2} - 11^{1/2} - \left(\cancel{\frac{1}{3}3^{3/2}} - 3^{1/2}\right)\right]$

$\boxed{\frac{\sqrt{11}}{2}\left[\frac{11}{3} - 1\right]}$

$\cancel{\frac{1}{3}3\sqrt{3}}$ $-\sqrt{3}$

d. $\int_1^3 \frac{e^{3/t}}{t^2} dt$

let $u = 3t^{-1}$
$du = -3t^{-2} dt$
$du = -\frac{3}{t^2} dt$

when $t = 1, u = 3$
$t = 3, u = 1$

$= -\frac{1}{3}\int_1^3 \frac{e^{3/t}}{t^2}(3dt) = -\frac{1}{3}\int_3^1 e^u du$

$= \frac{1}{3}\int_1^3 e^u du = \frac{1}{3}e^u\Big|_1^3 = \boxed{\frac{1}{3}(e^3 - e)}$